Mathematics

Single Correct Questions +4 | -1.00

1. If $I_1 = \int_0^1 e^{-x} cos^2 x \ dx$

$I_2 = \int_0^1 e^{-x^2} cos^2 x \ dx$ and

$I_3 = \int_0^1 e^{-x^3} dx$, then

(1) $I_3 > I_2 > I_1$
(2) $I_3 > I_1 > I_2$
(3) $I_2 > I_1 > I_3$
(4) $I_2 > I_3 > I_1$

2. The number of solutions of $\sin 3x = \cos 2x$, in the interval $\left(\frac{\pi}{2}, \pi\right)$ is :

(1) 4
(2) 3
(3) 2
(4) 1

3. If $f(x) = \sin^{-1}\left(\frac{2 \times 3^x}{1 + 9^x}\right)$, then $f'\left(-\frac{1}{2}\right)$ equals :

(1) $-\sqrt{3}\log_e 3$
(2) $-\sqrt{3}\log_e \sqrt{3}$
(3) $\sqrt{3}\log_e \sqrt{3}$
(4) $\sqrt{3}\log_e 3$

4. A plane bisects the line segment joining the points $(1, 2, 3)$ and $(-3, 4, 5)$ at right angles. Then this plane also passes through the point :

(1) $(1, 2, -3)$
(2) $(-3, 2, 1)$
(3) $(-1, 2, 3)$
(4) $(3, 2, 1)$

Space for rough use

5. If the system of linear equations
$x + ay + z = 3$
$x + 2y + 2z = 6$
$x + 5y + 3z = b$
has no solution, then :

(1) $a = 1, b \neq 9$

(2) $a \neq -1, b = 9$

(3) $a = -1, b = 9$

(4) $a = -1, b \neq 9$

6. If $\int \frac{2x+5}{\sqrt{7-6x-x^2}}dx = A\sqrt{7-6x-x^2} + B\sin^{-1}\left(\frac{x+3}{4}\right) + C$
(where C is a constant of integration), then the ordered pair (A, B) is equal to :

(1) $(-2, 1)$

(2) $(-2, -1)$

(3) $(2, -1)$

(4) $(2, 1)$

7. Let $f(x)$ be a polynomial of degree 4 having extreme values at $x = 1$ and $x = 2$
If $\lim_{x\to 0}\left(\frac{f(x)}{x^2} + 1\right) = 3$ then $f(-1)$ is equal to :

(1) $\frac{9}{2}$

(2) $\frac{3}{2}$

(3) $\frac{5}{2}$

(4) $\frac{1}{2}$

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8. The value of integral $\int_{\pi/4}^{3\pi/4} \frac{x}{1+sinx}dx$ is :

(1) $\pi\sqrt{2}$

(2) $2\pi\left(\sqrt{2}-1\right)$

(3) $\pi\left(\sqrt{2}-1\right)$

(4) $\frac{\pi}{2}\left(\sqrt{2}+1\right)$

9. A normal to the hyperbola, $4x^2-9y^2=36$ meets the co - ordinate axes $x \text{ and } y$ at A and B, respectively. If the parallelogram OABP (O being the origin) is formed, then the locus of P is :

(1) $4x^2-9y^2=121$

(2) $9x^2+4y^2=169$

(3) $9x^2-4y^2=169$

(4) $4x^2+9y^2=121$

10. Consider the following two statements :

Statement - P : The value of $\sin 120^0$ can be derived by taking $\theta = 240^0$ in the equation $2\sin\frac{\theta}{2}=\sqrt{1+\sin\theta}-\sqrt{1-\sin\theta}$

Statement - Q : The angles A,B,C and D of any quadrilateral ABCD satisfy the equation $\cos\left(\frac{1}{2}(A+C)\right)+\cos\left(\frac{1}{2}(B+D)\right)=0$

Then the truth values of P and Q are respectively :

(1) F,F

(2) F,T

(3) T,F

(4) T,T

11. The tangent to the circle $C_1 : x^2+y^2-2x-1=0$ at the point $(2,1)$ cuts off a chord of length 4 from a circle C_2 whose centre is $(3-2)$. The radius of C_2 is :

(1) 3

(2) 2

(3) $\sqrt{2}$

(4) $\sqrt{6}$

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12. Tangents drawn from the point $(-8,0)$ to the parabola $y^2 = 8x$ touch the parabola at P and Q. If F is the focus of the parabola, then the area of the triangle PFQ (in sq.units) is equal to :

(1) 24
(2) 32
(3) 48
(4) 64

13. If the mean of the data : $7, 8, 9, 7, 8, 7, \lambda, 8$ is 8, then the variance of this data is :

(1) 2
(2) 1
(3) $\frac{9}{8}$
(4) $\frac{7}{8}$

14. Let $A_n = \left(\frac{3}{4}\right) - \left(\frac{3}{4}\right)^2 + \left(\frac{3}{4}\right)^3 - .... + (-1)^{n-1}\left(\frac{3}{4}\right)^n$ and $B_n = 1 - A_n$. Then, the least odd natural number P, so that $B_n > A_n$ for all $n \geqslant p$ is :

(1) 9
(2) 7
(3) 5
(4) 11

15. Suppose A is any 3×3 non - singular matrix and $(A-3I)(A-5I) = O$, where $I = I_3$ and $O = O_3$. If $\alpha A + \beta A^{-1} = 4I$, then $\alpha + \beta$ is equal to :

(1) 8
(2) 12
(3) 7
(4) 13

16. The sides of a rhombus $ABCD$ are parallel to the lines, $x - y + 2 = 0$ and $7x - y + 3 = 0$. If the diagonals of the rhombus intersect at $P(1,2)$ and the vertex A (different from the origin) is on the y-axis, then the ordinate of A is:

(1) $\frac{7}{2}$
(2) $\frac{5}{2}$
(3) 2
(4) $\frac{7}{4}$

Space for rough use

17. An angle between the lines whose direction cosines are given by the equations, $l+3m+5n=0$ and $5lm-2mn+6nl=0$, is:

(1) $\cos^{-1}\left(\frac{1}{4}\right)$

(2) $\cos^{-1}\left(\frac{1}{3}\right)$

(3) $\cos^{-1}\left(\frac{1}{6}\right)$

(4) $\cos^{-1}\left(\frac{1}{8}\right)$

18. $\lim_{x\to 0}\frac{x\tan 2x-2x\tan x}{(1-\cos 2x)^2}$ equals:

(1) $-\frac{1}{2}$

(2) $\frac{1}{4}$

(3) $\frac{1}{2}$

(4) 1

19. If $f(x)$ is a quadratic expression such that $f(1)+f(2)=0$, and -1 is a root of $f(x)=0$, then the other root of $f(x)=0$ is:

(1) $-\frac{8}{5}$

(2) $-\frac{5}{8}$

(3) $\frac{5}{8}$

(4) $\frac{8}{5}$

20. The number of four letter words that can be formed using the letters of the word BARRACK is:

(1) 270

(2) 144

(3) 264

(4) 120

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21. Let $f(x)=\begin{cases}(x-1)^{\frac{1}{2-x}} & ,x>1, \quad x\neq 2\\ k & ,x=2\end{cases}$ The value of k for which f is continuous at $x=2$ is:

(1) 1

(2) e^{-2}

(3) e^{-1}

(4) e

22. Let $f:A\to B$ be a function defined as $f(x)=\dfrac{x-1}{x-2}$, where $A=R-\{2\}$ and $B=R-\{1\}$. Then f is:

(1) invertible and $f^{-1}(y)=\dfrac{2y-1}{y-1}$

(2) not invertible

(3) invertible and $f^{-1}(y)=\dfrac{2y+1}{y-1}$

(4) invertible and $f^{-1}(y)=\dfrac{3y-1}{y-1}$

23. The foot of the perpendicular drawn from the origin, on the line, $3x+y=\lambda(\lambda\neq 0)$ is P. If the line meets x-axis at A and y-axis at B, then the ratio $BP:PA$ is:

(1) $1:9$

(2) $1:3$

(3) $9:1$

(4) $3:1$

24. If $|z-3+2i|\leq 4$ then the difference between the greatest value and the least value of $|z|$ is:

(1) $4+\sqrt{13}$

(2) $2\sqrt{13}$

(3) $\sqrt{13}$

(4) 8

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25. It the position vectors of the vertices A, B and C of a ΔABC are respectively $4\hat{i}+7\hat{j}+8\hat{k},\ \ 2\hat{i}+3\hat{j}+4\hat{k}$ and $2\hat{i}+5\hat{j}+7\hat{k},$ then the position vector of the point, where the bisector of $\angle A$ meets BC is:

(1) $\frac{1}{4}\left(8\hat{i}+14\hat{j}+19\hat{k}\right)$

(2) $\frac{1}{2}\left(4\hat{i}+8\hat{j}+11\hat{k}\right)$

(3) $\frac{1}{3}\left(6\hat{i}+13\hat{j}+18\hat{k}\right)$

(4) $\frac{1}{3}\left(6\hat{i}+11\hat{j}+15\hat{k}\right)$

26. A tower T_1 of height $60\ m$ is located exactly opposite to a tower T_2 of height $80\ m$ on a straight road. From the top of T_1, if the angle of depression of the foot of T_2 is twice the angle of elevation of the top of T_2, then the width (in m) of the road betweeen the feet of the towers T_1 and T_2 is:

(1) $10\sqrt{3}$

(2) $20\sqrt{2}$

(3) $10\sqrt{2}$

(4) $20\sqrt{3}$

27. The curve satisfying the differential equation, $(x^2-y^2)dx+2xydy=0$ and passing through the point $(1,1)$ is:

(1) hyperbola

(2) circle of radius two

(3) a circle of radius one

(4) an ellipse

28. A player X has a biased coin whose probability of showing heads is P and a player Y has a fair coin. They start playing a game with their own coins and play alternately. The player who throws a head first is a winner. If X starts the game, and the probability of winning the game by both the players is equal, then the value of $'P'$ is:

(1) $\frac{1}{3}$

(2) $\frac{1}{4}$

(3) $\frac{1}{5}$

(4) $\frac{2}{5}$

Space for rough use

29. If a, b, c are in A.P. and a^2, b^2, c^2 are in G.P. such that $a < b < c$ and $a + b + c = \frac{3}{4}$, then the value of a is:

(1) $\frac{1}{4} - \frac{1}{2\sqrt{2}}$

(2) $\frac{1}{4} - \frac{1}{4\sqrt{2}}$

(3) $\frac{1}{4} - \frac{1}{3\sqrt{2}}$

(4) $\frac{1}{4} - \frac{1}{\sqrt{2}}$

30. The coefficient of x^{10} in the expansion of $(1+x)^2(1+x^2)^3(1+x^3)^4$ is equal to:

(1) 50

(2) 52

(3) 44

(4) 56

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