Mathematics

Single Correct Questions +4 | -1.00

1. Let $0 < x < \frac{1}{6}$ be a real number. When a certain biased dice is rolled, a particular face F occurs with probability $\frac{1}{6} - x$ and its opposite face occurs with probability $\frac{1}{6} + x$; the other four faces occur with probability $\frac{1}{6}$. Recall that opposite faces sum to 7 in any dice. Assume that the probability of obtaining the sum 7 when two such dice are rolled is $\frac{13}{96}$. Then, the value of x is:

   (A) $\frac{1}{8}$ (B) $\frac{1}{12}$ (C) $\frac{1}{24}$ (D) $\frac{1}{27}$

2. An office has 8 officers including two who are twins. Two teams, Red and Blue, of 4 officers each are to be formed randomly. What is the probability that the twins would be together in the Red team?

   (A) $\frac{1}{5}$ (B) $\frac{3}{7}$ (C) $\frac{1}{4}$ (D) $\frac{3}{14}$

3. Suppose Roger has 4 identical green tennis balls and 5 identical red tennis balls. In how many ways can Roger arrange these 9 balls in a line so that no two green balls are next to each other and no three red balls are together

   (A) 8 (B) 9 (C) 11 (D) 12

4. The number of permutations σ of $1, 2, 3, 4$ such that $|\sigma(i) - i| < 2$ for every $1 \le i \le 4$ is

   (A) 2 (B) 3 (C) 4 (D) 5

5. Let $f(x)$ be a degree 4 polynomial with real coefficients. Let z be the number of real zeroes of f. and e be the number of local extrema (i.e., local maxima or Minima) of f. Which of the following is a possible (z, e) pairs?

   (A) $(4,4)$ (B) $(3,3)$ (C) $(2,2)$ (D) $(0,0)$

6. A number is called a palindrome if it reads the same backward or forward. For example, 112211 is a palindrome. How many 6-digit palindromes are divisible by 495?

   (A) 10 (B) 11 (C) 30 (D) 45

7. Let A be a square matrix of real numbers such that $A^4 = A$. Which of the following is true for every such A ?

   (A) det$(A) \neq -1$

   (B) A must be invertiible.

   (C) A can not be invertiible.

   (D) $A^2 + A + I = 0$ where I denotes the identity matrix.

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8. Consider the real-valued function $h:\{0,1,...,100\} \to R$ such that $h(0)=5, h(100)=20$ and satisfying $h(i)=\frac{1}{2}(h(i+1)+h(i-1))$, for every $i=1,2,...,99$. Then, the value of $h(1)$ is :

(A) 5.15 (B) 5.5 (C) 6 (D) 6.15

9. An up-right path is a sequence of points $a_0=(x_0,y_0), a_1=(x_1,y_1), \cdots$ such that $a_{i+1}-a_i$ is either $(1,0)$ or $(0,1)$. The number of up-right paths from $(0,0)$ to $(100,100)$ which pass through $(1,2)$ is

(A) $3\binom{197}{99}$ (B) $3\binom{100}{50}$ (C) $2\binom{197}{98}$ (D) $3\binom{197}{100}$

10. Let $f(x)=\frac{1}{2}x\sin x-(1-\cos x)$. The smallest positive integer k such that $\lim_{x\to 0}\frac{f(x)}{x^k}\neq 0$ is :

(A) 3 (B) 4 (C) 5 (D) 6

11. Nine students in a class gave a test for 50 marks. Let $S_1 \le S_2 \le \ldots \le S_5 \le \ldots \le S_8 \le S_9$ denote their ordered scores. Given that $S_1=20$ and $\sum_{i=1}^{9} S_1 = 250$, let m be the smallest value that S_5 can take and M be the largest value that S_5 can take. Then the pair (m, M) is given by?

(A) $(20,35)$ (B) $(20,34)$ (C) $(25,34)$ (D) $(25,30)$

12. Let 10 red balls and 10 white balls be arranged in a straight line such that 10 each are on either side of a central mark. The number of such symmetrical arrangements about the central mark is

(A) $\frac{10!}{5!5!}$ (B) $10!$ (C) $\frac{10!}{5!}$ (D) $2.10!$

13. If $z=x+iy$ is a complex number such that $\left|\frac{z-i}{z+i}\right|<1$, then we must have

(A) $x>0$ (B) $x<0$ (C) $y>0$ (D) $y<0$

14. Let $S=\left\{x-y \mid x,y \text{ are real numbers with } x^2+y^2=1\right\}$. Then maximum number in the set S is

(A) 1 (B) $\sqrt{2}$ (C) $2\sqrt{2}$ (D) $1+\sqrt{2}$

15. In a factory, 20 workers start working on a project of packing consignments.They need exactly 5 hours to pack one consignment. Every hour 4 new workers joint the existing workforce. It is mandatory to would relive a worker after 10 hours. Then the number of consignments that would be packed in the initial 113 hours is

(A) 40 (B) 50 (C) 45 (D) 52

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16. Let $ABCD$ be a rectangle with its shorter side $a > 0$ units and perimeter $2s$ units. Let $PQRS$ be any rectangle such that vertices A, B, C and D respectively lie on the lines PQ, QR, RS and SP. Then the maximum area of such a rectangle $PQRS$ in square units is given by

(A) s^2 (B) $2a(s-a)$ (C) $\frac{s^2}{2}$ (D) $\frac{5}{2}a(s-a)$

17. The number of pairs of integers (x, y) satisfying the equation $xy(x+y+1) = 5^{2018}+1$ is

(A) 0 (B) 2 (C) 1009 (D) 2018

18. Let $p(n)$ be the number of digits when 8^n is written in base 6, and let $q(n)$ be the number of digits when 6^n is written in base 4. For example, 8^2 in base 6 is 144, hence $p(2) = 3$. Then $\lim\limits_{n\to\infty} \frac{p(n)q(n)}{n^2}$ equals:

(A) 1 (B) $\frac{4}{3}$ (C) $\frac{3}{2}$ (D) 2

19. For a real number α, let S_α denote the set of those real numbers β that satisfy $\alpha\sin(\beta) = \beta\sin(\alpha)$. Then which of the following statements is true?

(A) For any α, S_α is an infinite set

(B) S_α is finite set if and only if α is not an integer multiple of π

(C) There are infinitely many numbers α for which S_α is the set of all real numbers

(D) S_α is always finite

20. If $A = \begin{pmatrix} 1 & 1 \\ 0 & i \end{pmatrix}$ and $A^{2018} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, then $a+d$ equals :

(A) $1+i$ (B) 0 (C) 2 (D) 2018

21. Let $f : R \to R$ and $g : R \to R$ be two functions. Consider the following two statements :
$P(1)$: If $\lim\limits_{x\to 0} f(x)$ exists and $\lim\limits_{x\to 0} f(x)g(x)$ exists, then $\lim\limits_{x\to 0} g(x)$ must exist.
$P(2)$: If f, g are differentiable with $f(x) < g(x)$ for every real number x, then $f'(x) < g'(x)$ for all x
Then, which one of the following is a correct statement ?

(A) Both $P(1)$ and $P(2)$ are true.

(B) Both $P(1)$ and $P(2)$ are false.

(C) $P(1)$ is true and $P(2)$ is false.

(D) $P(1)$ is false and $P(2)$ is true.

22. The number of solutions of the equation $sin(7x) + sin(3x) = 0$ with $0 \le x \le 2\pi$ is :

(A) 9 (B) 12 (C) 15 (D) 18

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23. A bag contains some candies, $\frac{2}{5}$ of them are made of white chocolate and remaining $\frac{3}{5}$ are made of dark chocolate. Out of the white chocolate candies, $\frac{1}{3}$ are wrapped in red paper, the rest are wrapped in blue paper. Out of the dark chocolate candles, $\frac{2}{3}$ are wrapped in red paper, the rest wrapped in blue paper. If a randomly selected candy from the bag is found to be wrapped in red paper, then what is the probability that it is made up of dark chocolate ?

(A) $\frac{2}{3}$ (B) $\frac{3}{4}$ (C) $\frac{3}{5}$ (D) $\frac{1}{4}$

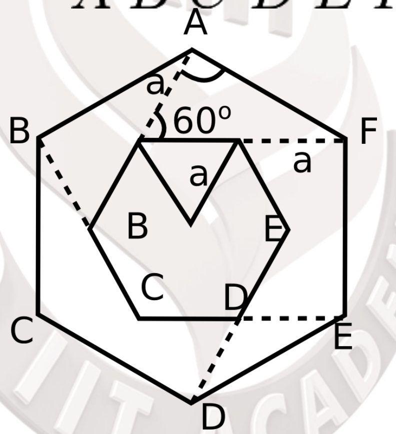
24. A party is attended by twenty people. In any subset of four people, there is at least one person who knows the other three (we assume that if X knows Y, then Y knows X). Suppose there are three people in the party who do not know each other. How many people in the party know everyone ?

(A) 16
(B) 17
(C) 18
(D) Cannot be determined from the given data.

25. The sum of all natural numbers a such that $a^2 - 16a + 67$ is a perfect square is :

(A) 10 (B) 12 (C) 16 (D) 22

26. The sides of a regular hexagon $ABCDEF$ are extended by doubling them (for example, BA extends to BA' with $BA' = 2BA)$ to form a bigger regular hexagon $A'B'C'D'E'F'$ as in the figure



Then the ratio of the areas of the bigger to the smaller hexagon is:

(A) 2 (B) 3 (C) $2\sqrt{3}$ (D) π

27. Between 12 noon and 1 PM, there are two instants when the hour hand and the minute hand of a clock are at right angles. The difference in minutes between these two instants is:

(A) $32\frac{8}{11}$ (B) $30\frac{8}{11}$ (C) $32\frac{5}{11}$ (D) $30\frac{5}{11}$

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28. For which values of θ, with $0 < \theta < \frac{\pi}{2}$, does the quadratic polynomial in t given by $t^2 + 4t\cos\theta + \cot\theta$ have repeated roots ?

(A) $\frac{\pi}{6}$ or $\frac{5\pi}{18}$ (B) $\frac{\pi}{6}$ or $\frac{5\pi}{12}$ (C) $\frac{\pi}{12}$ or $\frac{5\pi}{18}$ (D) $\frac{\pi}{12}$ or $\frac{5\pi}{12}$

29. Let α, β, γ be complex numbers which are the vertices of an equilateral triangle. Them, we must have :

(A) $\alpha + \beta + \gamma = 0$

(B) $\alpha^2 + \beta^2 + \gamma^2 = 0$

(C) $\alpha^2 + \beta^2 + \gamma^2 + \alpha\beta + \beta\gamma + \gamma\alpha = 0$

(D) $(\alpha - \beta)^2 + (\beta - \gamma)^2 + (\gamma - \alpha^2) = 0$

30. Assume that n copies of unit cubes are glued together side by side to form a rectangular solid block. If the number of unit cubes that are completely invisible is 30, then the minimum possible value of n is :

(A) 204 (B) 180 (C) 140 (D) 84

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